

Chapter Three: Dimensional Analysis

Dimensional analysis is a very handy method for changing measurements from one dimension into another. Sometimes referred to as the factor-label method, dimensional analysis not only permits one to easily modify dimensions, but it also often provides a method of solving certain problems by providing a dimension pattern to the desired result. The method provides students with a strategy for multiplying or dividing measurements by conversion factors to yield the required dimension requested. The methodology works on simple as well as quite sophisticated problems, too.

Initially students believe that dimensional analysis is time consuming tedious and unnecessary. However, as students work with more involved problems with complicated dimensions they quickly find the dimensional analysis to be an invaluable problem solving tool. In addition, dimensional analysis can be an aid in remembering formulas or equations which have been forgotten.

An example encountered in algebra class is the speed or rate of motion equation. Is the equation for speed distance divided by time or time divided by distance? If one supplies the units: $r = \text{miles/hr}$ or $r = \text{hr/miles}$ the answer becomes clear since speed is commonly given in miles/hr. The key to using dimensional analysis is to realize that units or dimensions can be added, subtracted, multiplied, or divided just as numbers. For example,

$$9 \text{ ft}^2 + 13 \text{ ft}^2 = 22 \text{ ft}^2 \quad \text{or} \quad 37.3 \text{ cm} - 31.4 \text{ cm} = 5.9 \text{ cm}$$

Only similar dimensions may be added or subtracted as you are undoubtedly aware.

The operations for multiplication, division, and finding the root are different in that measurement with different dimensions can be combined with these operations. To determine the area of a rectangular room, one must multiply the length and width of the room in the same units to obtain the area. A 9.0 ft by 12.0 ft room has an area of 108 square feet or 110 ft^2 squared using significant digits.

The speed of an object is found by dividing distance by time, $r = d/t$. If a distance of 75 miles were traveled in 3.0 hours, then the average speed would be 25 miles/hour. In other words the dimensions were also divided to obtain the appropriate dimension for speed.

Another example, which combines several operations, is find momentum (p) by multiplying mass and velocity, $p = mv$; thus, if a mass of 2.0 kg is moving at 12 m/sec then the momentum of the object is $24 \text{ kg} \cdot \text{m/sec}$. The dimensions are multiplied and the appropriate momentum unit becomes kilogram · meters/second.

Dimensions can cancel when like dimensions are divided. The specific gravity of an object is determined by dividing the density of an object by the density of water. The density of lead 11.35 gm/cc divided by water 1.00 gm/cc yields a specific gravity of 11.35 with no units for the units canceled out.

Strategy in Completing Conversion Problems

Arguably the most useful form of dimensional analysis is dealing with unit conversion. The method is quite simple and almost infallible if followed carefully. Suppose one wanted to convert 6.0 feet to inches. The idea is to choose the proper conversion factor $1 \text{ foot} = 12 \text{ inches}$, and multiply 6.0ft by the conversion factor

written as a ratio or fraction. The dimensions designate which factor is in the denominator and the numerator. For example,

$$6.0 \text{ ft} \times 12 \text{ in.} / 1 \text{ ft} = 72 \text{ in.}$$

Note the feet units canceled yielding inches.

The ratio, or fraction, $12 \text{ in}/1 \text{ ft}$ is equal to one since the numerator and denominator are equivalent. Therefore, since one is the multiplication identity, then multiplying by one does not change the value of the quantity, only the dimensions.

Another example involves changing 65.0 miles to centimeters: One method is to change miles to feet and feet to centimeters. Therefore, one will need the two conversion equations of $1 \text{ miles} = 5280 \text{ feet}$ and $1 \text{ foot} = 30.48 \text{ cm}$.

$$65.0 \text{ miles} \times 5280 \text{ ft}/1 \text{ mi} \times 30.48 \text{ cm}/1 \text{ ft} = 1.05 \times 10 \text{ to the } 7^{\text{th}} \text{ cm}$$

Note that the miles and feet units canceled out and cm were left as the desired units. Each of the two factors of $5280 \text{ ft}/1 \text{ mi}$ and $30.48 \text{ cm}/1 \text{ ft}$ were both equal to one as required.

Suppose one wanted to convert the speed of an object of 55.0 miles per hour to a more useful unit of ft/sec. This can be done quite conveniently and quickly using this method. In this case, the miles must be changed to feet and the hours must be changed to seconds. Conversion equations: $5280 \text{ ft} = 1 \text{ mile}$ $60 \text{ minutes} = 1 \text{ hour}$ $60 \text{ seconds} = 1 \text{ minute}$

$$\text{Example: } 55.0 \frac{\text{mi}}{\text{hr}} \times \frac{5280 \text{ ft}}{1 \text{ mi}} \times \frac{1 \text{ hr}}{60 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ sec}} = 80.7 \frac{\text{ft}}{\text{sec}}$$

Note that the miles in the numerator canceled leaving feet and that hr and min in the denominator canceled leaving sec in the denominator thus ft/sec, is the desired unit, is all that remains.

The dimensions help the student multiply and divide at the proper times to achieve the desired conversion. The method is even more successful on dimensions and conversions which are strange and unfamiliar.

REMEMBER, in completing conversion problems, the number of significant figures in the result is the same as the number of significant figures in the original measurement to be converted.

Reducing Derived Units

Derived units such as energy and force are composed of two or more elemental units. The dimensions help one to make sense of the more complicated derived units, and the method works as above.

Example: The SI units of force are the Newton. The Newton can be written as follows

$$\text{Newton} = \frac{\text{kg} \cdot \text{m}}{\text{sec}^2}$$

This is read "kilogram meters per second squared" and is easier to calculate with.

Example: The Joule is the unit of energy. The joule is a derived unit.

$$\text{Joule} = \frac{\text{kg} \cdot \text{m}^2}{\text{sec}^2}$$

This is read "kilogram meter squared per second squared"

When units are treated in a manner similar to the way algebraic quantities are treated, then some unusual cases can be found. If the square root of a quantity is taken, then the result may have a unit such as square root of a dimension.

Example: Square root of 36 cm = 6.0 $\sqrt{\text{cm}}$

Often the exponent of $\frac{1}{2}$ is used as an exponent to represent square root. Therefore, the above answer is written 6.0 $\text{cm}^{\frac{1}{2}}$. This means the square root of centimeters.

Another common example is the negative exponent "-1". The number 10 to the -1 is the reciprocal of 10 or one tenth (1/10). So the unit 1/sec (per sec) is often written as sec^{-1} . The unit for frequency, the hertz, is fundamentally sec^{-1} (or 1/sec).

Exercises: Units and Unit Conversions

In the following equations, substitute the appropriate units for the symbols and find the missing left-hand unit.

s in meters (m) a in m/sec^2	t in sec m (mass) in kg	v in m/sec F in newtons or $\text{kg} \cdot \text{m}/\text{sec}^2$
1. $? = vt$		7. $? = ma$
2. $? = at$		8. $? = mv/F$
3. $? = at^2$		9. $? = 4(\pi)^2 \text{ms}/t^2$
4. $? = v/a$		10. $? = Ft^2/4\pi^2 m$
5. $?^2 = as$		11. $? = 2(s-vt)/t^2$
6. $? = mv^2/s$		12. $? = ms/t$

Convert the following quantities to the required units, using dimensional analysis. You will probably need the conversion equation sheets at the back of this booklet.

13. 88 ft/sec to mi/hr
14. 50 light years to kilometers
15. 140,000 miles to kilometers
16. 2000 cubic feet to cubic yards
17. 0.75 cubic meters to cubic cm
18. 68 inches to cm
19. 50 miles to kilometers
20. 12 square miles to square cm
21. 2,000 cubic feet to gallons
22. 1 day to seconds
23. 32 mi/hr to m/sec
24. 250 ml to cubic cm
25. 3.5 years to minutes

Chapter 4: Estimation

Many times in physics, it is useful to have a "ball park" idea of what the solution to a numerical problem will be before performing any exact calculations. Students often find estimation (order of magnitude) calculations difficult to perform because they are commonly made without knowing any information about the "actual" answer. In order to combat this difficulty, we will adhere to the "practice makes perfect" adage and hone your estimation skills with Fermi questions.

Enrico Fermi, the famed Italian physicist, was legendary for being able to figure out things in his head, using information that initially seemed too meager for a quantitative result. He used a process of "zeroing in" on problems by saying that the value in question was certainly larger than one number and less than some other amount. He would proceed through a problem in that fashion and, in the end, have a quantified answer within identified limits.

The questions named after Fermi emphasize estimation, numerical reasoning, communicating in mathematics, and questioning skills. Fermi questions encourage multiple approaches, emphasize process rather than "the answer", and promote non-traditional problem solving strategies. In a Fermi question, the goal is to get an answer to an order of magnitude (typically a power of ten) by making reasonable assumptions about the situation, not necessarily relying upon definite knowledge for an "exact" answer.

A Fermi question is posed with limited information given. (How many water balloons would it take to fill the school gymnasium?) The question requires that the student ask many more questions in order to work toward an estimated answer. (How big is a water balloon? What are the approximate dimensions of the gym? What measurements must be estimated to find the dimensions of the gym? ... and the list goes on) In order to see how these questions can be answered, two "typical" Fermi questions are posed and solved below. WARNING: Fermi questions are not really typical and Fermi questions do not have single solutions. These two examples are provided simply for illustrative purposes.

Example 1: HOW MANY PIANO TUNERS ARE IN NEW YORK CITY?

How might one figure out such a thing?? Surely the number of piano tuners in some way depends on the number of pianos. The number of pianos must connect in some way to the number of people in the area. Approximately how many people are in New York City? 10,000,000. Does every individual own a piano? No. Would it be reasonable to assert that "individuals don't tend to own pianos; families do? Yes. About how many families are there in a city of 10 million people? Perhaps there are 2,000,000 families in NYC. Does every family own a piano? No. Perhaps one out of every five does. That would mean there are about 400,000 pianos in NYC. How many piano tuners are needed for 400,000 pianos? Some people never get around to tuning their piano; some people tune their piano every month. If we assume that "on the average" every piano gets tuned once a year, then there are 400,000 "piano tunings" every year. How many piano tunings can one piano tuner do? Let's assume that the average piano tuner can tune four pianos a day. Also assume that there are 200 working days per year. That means that every tuner can tune about 800 pianos per year. How many piano tuners are needed in NYC? The number of tuners is approximately $400,000/800$ or 500 piano tuners.

Try it yourself. Use different assumptions for the various factors. It is unlikely that you can justify an answer greater than a factor of 10 or smaller than a factor of 10 from the number originally obtained; that is to say, there are probably not more than 5000 tuners and surely no less than 50. Thus the answer obtained is good to within an "order of magnitude".

Example 2: HOW MANY JELLY BEANS FILL A ONE-LITER BOTTLE? (One liter is equal to 1000 cubic centimeters.)

What is the approximate size a jelly bean? An examination of a jelly bean reveals that is approximately the size of a small cylinder that measures about 2 cm long by about 1.5 cm in diameter. Do jelly beans "completely fill the liter bottle"? The irregular shape of jelly beans result in them not being tightly packed; approximately 80% of the volume of the bottle is filled. The number of jelly beans is the occupied volume of the jar divided by the volume of a single jelly bean. Number of beans = (Occupied Volume of Jar)/(Volume of 1 Bean). The volume of one jelly bean is approximated by the volume of a small cylinder 2 cm long and 1.5 cm in diameter. Volume of 1 Jelly Bean = $h(\pi)(d/2)^2 = 2\text{cm} \times 3(1.5\text{cm}/2)^2 = 27/8$ cubic centimeters. Thus the approximate number of beans in the jar is Number of beans = $(.80 \times 1000 \text{ cubic centimeters})/(27/8 \text{ cubic centimeters}) = \text{approx } 240$ jelly beans

Exercises: Estimation

Perform the following Fermi calculations and express your final answer in scientific notation.

1. What is the mass in kilograms of the student body in your school?
2. How many golf balls will fill in a suitcase?
3. How many gallons of gasoline are used by cars each year in the United States?
4. How high would the stack reach if you piled one trillion dollar bills in a single stack?
5. How many hairs are on your head?
6. How fast does it grow in miles per hour?
7. If your life earnings were doled out to you at a certain rate per hour for every hour of your life, how much is your time worth?
8. How many hot dogs will be eaten at all major league baseball games during one full season?

Chapter 5: Logarithms

Logarithm, which has been symbolized by *log*, is only a fancy name for exponents. A logarithm is an exponent therefore; all the mathematical rules which work for exponent ultimately apply to logarithms. The concept of logarithms used extensively throughout physics, chemistry, and other physical sciences. Some examples of logarithms in action: the decibel rating of sounds is a logarithm, the pH of solutions in chemistry is a logarithm, the slide rule computing device was based on logarithms and the numerical value of the Richter scale for earthquakes is a logarithms, just to mention a few. Thus, it is imperative that student develop a working relationship and understanding of logarithms.

The Meaning of Logarithm

The basis for understanding logarithms is rooted in using exponents. Since 2^3 equals 8, then the exponent that 2 must be raised to in order to yield 8 is 3. Also, since 10^2 equals 10 x 10 or 100 then the exponent that 10 must be raised to in order to yield 100 is 2. These two exponents, 3 and 2, are called logarithms. In general in the equation $y = b^x$, b is the base, and the exponent x is the logarithm of the number y to the base b .

The logarithm of a number is the exponent or power to which a base must be raised to give that number.

Usually the number 1 is not chosen as the base of a logarithm since 1 to any power is equal to 1. The base b is also restricted to be nonnegative.

Definition of Logarithm

If b and N are positive numbers ($b \neq 1$), then $\log_b N = k$ iff $b^k = N$

Note: <iff> means "if and only if"

Examples:

$\log_6 36 = 2$
$\log_2 16 = 4$
$\log_{10} 1000 = 3$
$\log_2 25 = 2$

In order to shorten the notation for logarithms, if \log is written without a base notation, then it is understood to be base 10. In other words, $\log_{10} N = \log N$

Logarithms and Their Properties

Suppose you wanted to find the $\log 215$. Simply press <LOG> on your calculator, enter 215, then press <ENTER>. The result is 2.33243846, or the logarithm of 215. Since $\log 215 = 2.33243846$, then 10 (the understood base) raised to the 2.33243846 power will yield 215.

$$10^{2.33243846} = 215$$

The exponent 2.33243846 has two parts. The 2 before the decimal is called the characteristic and indicates the decimal placement (215 is 2.15×10^2). The 0.33243846 is called the mantissa and it indicates the sequence of numbers, namely 215, when the inverse function of the logarithm is applied to the mantissa. The mantissa is an irrational number whose inverse only approximates the original number. Either rounding off or

using significant figures is appropriate in logarithms. Placing 0.33243846 on your calculator and pressing the inverse log button (10^x), the result will be 215.

Several other properties of logarithms are apparent since they are equivalent to exponents. If we let b be the base of a logarithmic function with $b > 0$ but $b \neq 1$ and let M and N be positive numbers then the following laws apply to logarithms:

1. $\log_b MN = \log_b M + \log_b N$ (if one multiplies numbers with the same base, retain the base and add exponents)
2. $\log_b M/N = \log_b M - \log_b N$ (if one divides numbers with the same base, retain the base and subtract the exponents)
3. $\log_b M^k = k \log_b M$ (if one raises a power to a power, retain the base and multiply the exponents)
4. $\log_b 1/N = -\log_b N$ (use rule 2 and the fact that $\log 1 = 0$)

Another important characteristic of logarithms comes from the fact that a negative exponent means reciprocal. For example $b^{-3} = 1/b^3$. Just as in scientific notation, if one has a negative logarithm (exponent), then the reciprocal function is applicable. A large negative logarithm represents a very small number.

Example: If $\log M = -4.24394$, then $M = 0.000057025$

Exercises: Logarithms

Write the following numbers in exponential notation and determine “?”

1. $\log_5 125 = ?$
2. $\log_3 81 = ?$
3. $\log_2 1 = ?$
4. $\log_2 1/8 = ?$
5. $\log 10\,000 = ?$
6. $\log 0.0001 = ?$

Write the following exponential equations in *log* notation.

7. $7^2 = 49$
8. $5^3 = 125$
9. $10^4 = 10\,000$
10. $10^{3.524} = 3341.95$
11. $2^{-5} = 1/32$
12. $10^{-.30103} = 5$

Find the following logarithms using your calculator. Keep the same number of significant figures in the answer as in the original number.

13. $\log 315$
14. $\log 0.0076$
15. $\log 93,500,000$
16. $\log 6.67 \times 10^{-11}$

Find the anti-logarithm (inverse log) with your calculator for the following:

17. $\log x = 3.7226$
18. $\log x = 2.1303$
19. $\log x = -3.8297$
20. $\log x = 1.30103$

Chapter 6: Data Analysis

In physics, relationships between physical quantities are determined experimentally and expressed mathematically. As such, it is important for any budding scientist to master the skills required to analyze physical data and determine what, if any, mathematical model relates the data. In this section, you will learn how to analyze data using a method known in math and science as regression analysis.

Computer Based Regression Analysis (Microsoft Excel)

ENTERING DATA

1. In order to create graphs you must first enter your data into a spreadsheet. To do this open MS Excel and type your independent variable in the first column and your dependent variable in the second column. On the first line in each column, label the quantity and state the units. (see below)

t(sec)	x(m)
0	0
1	5
2	20
3	45
4	80
5	125

PLOTTING DATA POINTS

2. Once you have entered the data you are ready to plot the data on the graph. To do so, use your mouse to highlight the two columns of data, excluding the labels.

t(sec)	x(m)
0	0
1	5
2	20
3	45
4	80
5	125

3. Once the data is highlighted as shown above, click the pull down menu INSERT at the top of the window and choose CHART from the menu. This will open what is called the "Chart Wizard" which will allow you to choose some of the basic features of your plot.
4. Once the Chart Wizard has popped up, click XY SCATTER from the chart type list and choose the default chart sub-type as seen below and then click NEXT.

